EQUAZIONI LAGRANGE

$\downarrow$

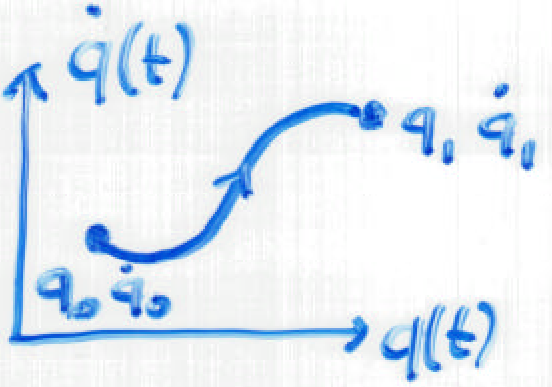
VARIABILI DI STATO

STATI SE n giunti $q_i(t)$
SONO $2n$

$$\underline{x} = \begin{bmatrix} \underline{q}(t) \\ \dot{\underline{q}}(t) \end{bmatrix} \qquad \underline{x} \to \underline{\xi}$$

$$\underline{\xi}(t) = T\underline{x}$$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} \rightarrow \left.\begin{matrix} \xi_1 = x_1 + x_2 \\ \xi_2 = x_1 - x_2 \end{matrix}\right\} \quad T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}; t)$$

$$\underline{y} = \underline{g}(\underline{x}, \underline{u}; t)$$

x Stati
u ingressi
y uscite

$$\begin{cases} \dot{\underline{x}} = \underline{f}(\underline{x}(t), \underline{u}(t)) \\ \underline{y} = \underline{g}(\underline{x}(t), \underline{u}(t)) \end{cases}$$

NON LIN
TEMPO INVARIANTI

$$\begin{cases} \dot{\underline{x}} = A(t)\,\underline{x}(t) + B(t)\,\underline{u}(t) \\ \underline{y} = C(t)\,\underline{x}(t) + D(t)\,\underline{u}(t) \end{cases}$$

LIN
TEMPO
VARIANTI

$$\begin{cases} \dot{\underline{x}}(t) = A\,\underline{x}(t) + \underline{B}\,\underline{u}(t) \\ \underline{y}(t) = C\,\underline{x}(t) + D\,\underline{u}(t) \end{cases}$$

LTI

$$\begin{cases} \dot{\underline{x}}(t) = A\,\underline{x}(t) + B\,\underline{u}(t) \\ \underline{y}(t) = C\,\underline{x}(t) \end{cases}$$

$$H(\underline{q}(t))\,\ddot{\underline{q}}(t) + C(\underline{q}, \dot{\underline{q}})\,\dot{\underline{q}} + \underline{g}(\underline{q}) = \underline{\tau}(t)$$

$$\Downarrow$$

$$\ddot{q}(t) = H^{-1}(\underline{q}(t))\left[-C(\underline{q}, \dot{\underline{q}})\,\dot{\underline{q}}(t) - \underline{g}(\underline{q}) + \tau(t)\right]$$

$$\underline{x}_1 = \underline{q}(t) \qquad \underline{x}(t) = \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} \qquad ★★★$$

$$\underline{x}_2 = \underline{\dot{q}}(t)$$

$$\underline{\xi}(t) = \begin{bmatrix} q_1 \\ \dot{q}_1 \\ \vdots \\ q_n \\ \dot{q}_n \end{bmatrix}$$

$$\underline{\dot{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$$

$$\dot{x}_1(t) = \dot{q}(t) = x_2(t)$$

$$\dot{x}_2(t) = \ddot{q}(t) = -H^{-1}C\dot{q}\ \boxed{-H^{-1}g + H^{-1}\tau}\ ★$$

$$-H^{-1}C(\underline{x}_1, \underline{x}_2)\underline{x}_2$$

$$-H^{-1}(\underline{x}_1(t))C(\underline{x}_1, \underline{x}_2)\underline{x}_2$$

NON LINEARE

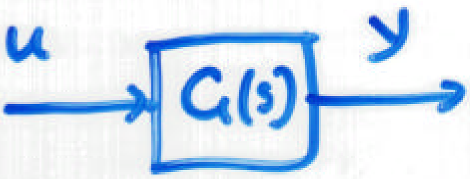
$$★\ -H^{-1}(\underline{x}_1)\underline{g}(x_1) + H^{-1}(\underline{x}_1)\underline{\tau}(t) \equiv \underline{u}_c$$

COME PROGETTIAMO UN CONTROLLORE?

— o —

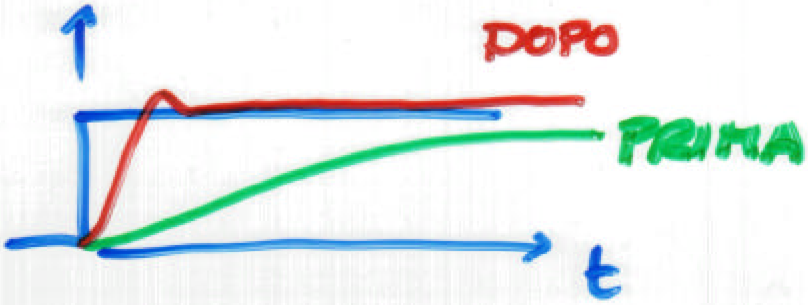
COSA AVETE STUDIATO?

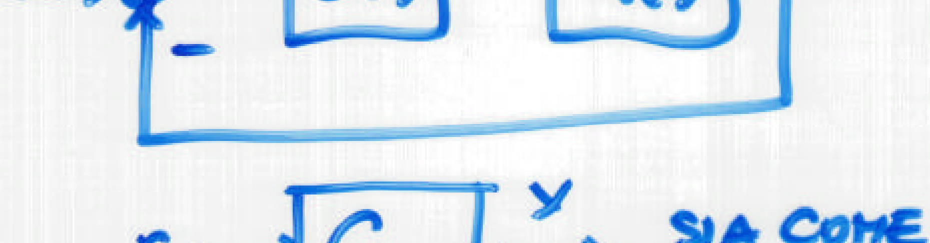
DATA LA FDT ...

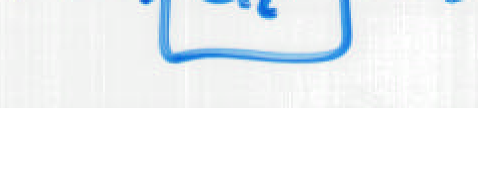


SISO

PROGETTARE UN CONTROLLO CHE ABBIA CERTE SPECIFICHE







SIA COME VOGLIO IO

ROBOT : NONLINEARE
MIMO

$\dot{x}_1 = x_2$ LINEARE

$\dot{x}_2 = -H^{-1} C \dot{q} - H^{-1} g + H^{-1} \tau$

COME PUÒ DIVENTARE APPROX LINEARE ?

- $C \dot{\underline{q}} \simeq \underline{\varepsilon}$ TRASCURABILE

VELOCITÀ $\dot{q}_i$ SONO BASSE
ALLORA $\rightarrow \underline{\varepsilon}$

$\dot{x}_2 = -H^{-1} g + H^{-1} \tau$

- RIUSCIAMO A COMPENSARE LA GRAVITÀ

$\underline{g} \rightarrow \underline{0}$

$\dot{x}_2 = H^{-1} \tau = H^{-1}(\underbrace{\underline{q}(t)}_{\underline{x}_1(t)}) \underline{\tau}(t)$

- SE $H(q(t)) = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$ PIENA E $q(t)$ DIPENDENTE

$$\Downarrow$$

$$H^* = \begin{bmatrix} \square & & \phi \\ & \square & \\ \phi & & \square \end{bmatrix} \quad \text{E } q(t) \text{ INDIPENDENTE}$$

$$H^* = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_2 & 0 \\ 0 & 0 & H_3 \end{bmatrix}$$

$$(H^*)^{-1} = \begin{bmatrix} \frac{1}{H_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{H_3} \end{bmatrix}$$

$$\dot{x}_2 = (H^*)^{-1} u$$

- LINEARIZZAZIONE STRUTTURALE
- DISACCOPPIAMENTO

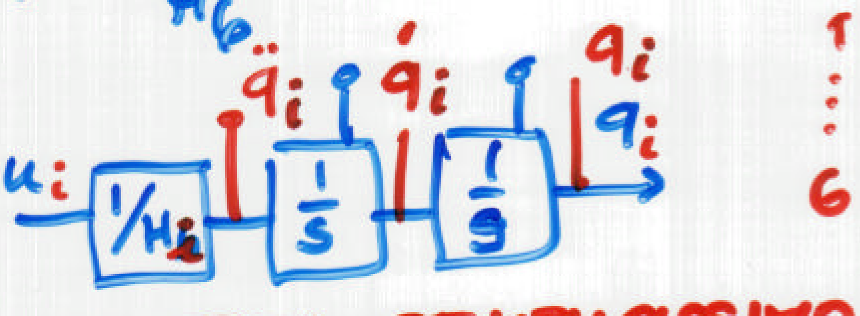
$$\begin{cases} \dot{\underline{x}}_1 = \underline{x}_2 \\ \dot{\underline{x}}_2 = (H^*)^{-1} u \end{cases}$$

$$\begin{cases} \dot{x}_{11} = x_{21} \\ \dot{x}_{21} = \dfrac{1}{H_1} u \end{cases}$$

$$\begin{cases} \dot{x}_{12} = x_{22} \\ \dot{x}_{22} = \dfrac{1}{H_2} u \end{cases}$$

$$\vdots$$

$$\begin{cases} \dot{x}_{16} = x_{26} \\ \dot{x}_{26} = \dfrac{1}{H_6} u \end{cases}$$



SISTEMA SEMPLICISSIMO
MA INSTABILE

PIANIFICAZIONE

- OBBIETTIVO
- COMPITO / TASK
- OPERAZIONE
- MOSSA / MOTION ... MOVE
- TRAIETTORIA / CAMMINO
  TRAJECTORY / PATH
- CONTROLLO / RIFERIMENTO

PIANIFICAZIONE

$T = 1\,ms$

riferimenti x il controllo

CAMMINO = GEOMETRIA SENZA TEMPO

TRAIETTORIA = CAMMINO + TEMPO

VINCOLI TECNOLOGICI

ROBOTICA INDUSTRIALE

- MOTORI
- STRUTTURE
- OGGETTI TRASPORTATI

- MOTORI : VELOCITA' MAX
  : ACCELERAZIONI MAX
  : COPPIE MAX
  : RISCALDAMENTO
  ⋮

- STRUTTURE : FREQUENZA DEI SEGNALI

- OGGETTI TRASPORTATI
  : ACCELERAZIONI CENTRIFUGHE