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Prova

CIN DIRETTA POSIZIONE

- Multicorpo n corpi
- Sistemi Riferimento $n+1$
- T_i^{i-1}
- $T_n^0 = T_1^0 \cdots T_n^{n-1} = \begin{bmatrix} R & \underline{t} \\ 0^T & 1 \end{bmatrix}$

$\underline{p} = \underline{f}(\underline{q})$ $\quad q_i$? parametri di DH

angoli $\leftrightarrow$ giunto è rotoidale

spostamenti $\leftrightarrow$ giunto prismatico

$\underline{q} = n \times 1$

$\underline{p} = \begin{bmatrix} \underline{x} \\ \underline{\alpha} \end{bmatrix} \Rightarrow \underline{x}(t) = \underline{t}(\underline{q}(t))$

$\underline{\alpha}$? $\quad R(\underline{q}(t))$ contiene 3 parametri indipendenti

ϕ, θ, ψ angoli di Eulero

$\theta_x \ \theta_y \ \theta_z$
angoli di
roll pitch yaw

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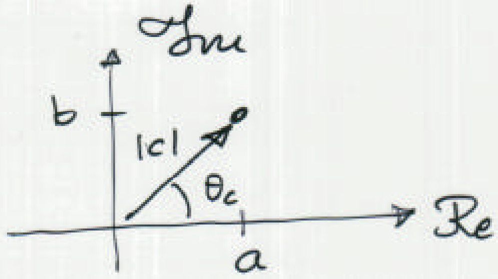
QUATERNIONI

$i = \sqrt{-1}$

$c = a + jb \qquad \underline{a + ib}$



$c \rightarrow |c| = 1 \quad = r = r_1 + i r_2$

$r_1^2 + r_2^2 = 1$

$rc \; ? \; = \qquad |rc| = |c| \quad$ se $\; |r| = 1$

$\underline{\angle rc} = \underline{\angle c} + \underline{\angle r}$

parte vettoriale

$$h = \boxed{h_0} + \boxed{h_1 i + h_2 j + h_3 k}$$

$$i^2 = -1 \qquad j^2 = -1 \qquad k^2 = -1$$

$ij = k = -ji$ anticommutativo

$$(a_1^2 + b_1^2)(a_2^2 + b_2^2) = c_1^2 + c_2^2$$

$$a^2 b^2 = c^2$$

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$h \rightarrow$ quaternione unitario

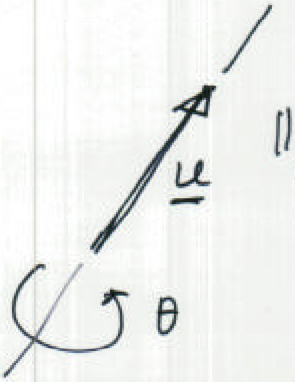
$$\|h\| = 1 \qquad \|h\| = \sqrt{h_0^2 + h_1^2 + h_2^2 + h_3^2}$$

↳ OPERATORE DI ROTAZIONE !

$h_1 = u_1 \sin \frac{\theta}{2}$

$h_2 = u_2 \sin \frac{\theta}{2}$

$h_3 = u_3 \sin \frac{\theta}{2}$



$\|\underline{u}\| = 1$

$h_0 = \cos \frac{\theta}{2}$

$$\|h\| = \|\underline{u}\| \underbrace{\left(\cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)\right)}_{=1}$$

$$1 \quad = \quad =1 \quad \cdot$$

Rotazione intorno asse x di 90°

asse x $\rightarrow$ $\underline{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\theta = 90 \qquad \frac{\theta}{2} = 45$

$$\left.\begin{aligned} h_0 &= \frac{\sqrt{2}}{2} \\ h_1 &= 1 \cdot \frac{\sqrt{2}}{2} \\ h_2 &= 0 \\ h_3 &= 0 \end{aligned}\right\} \quad \underline{h} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0, 0\right)$$

h & g $\quad h \cdot g$?

$hg \neq gh$ la formula è in (B.12)

matrici di rotazione $\qquad$ quaternioni

$R_1 R_2 R_3 \dots = R \quad \longleftrightarrow \quad h_1 h_2 h_3 \dots = h$

$\downarrow \; \downarrow \; \downarrow \qquad \downarrow$

$h_1 \; h_2 \; h_3 \qquad h$

CONIUGATO

$R \Leftrightarrow R^T \qquad h \Leftrightarrow h^*$

$(h_0 \, h_1 \, h_2 \, h_3) \Leftrightarrow (h_0 \, -h_1 \, -h_2 \, -h_3)$

$\underline{x}$ vettore 3×1

$h_x = (0, x_1, x_2, x_3)$

$R\,\underline{x} = \underline{y}$

$h\,x\,h^*$

h

CIN DIRETTA DI POSIZIONE

$$\underline{p} = \begin{bmatrix} \underline{x} \\ \underline{\alpha} \end{bmatrix}$$

$\underline{\alpha}$? se uso i quaternioni

CINEMATICA VELOCITÀ = calcolo dello Jacobiano

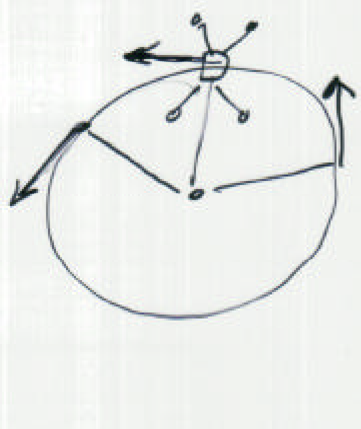
$$\dot{\underline{p}}(t) = \begin{bmatrix} \dot{\underline{x}} \\ \dot{\underline{\alpha}} \end{bmatrix} = J(\underline{q}(t))\,\dot{\underline{q}}(t)$$

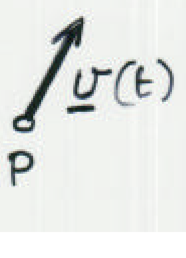
JACOBIANO ANALITICO

" GEOMETRICO

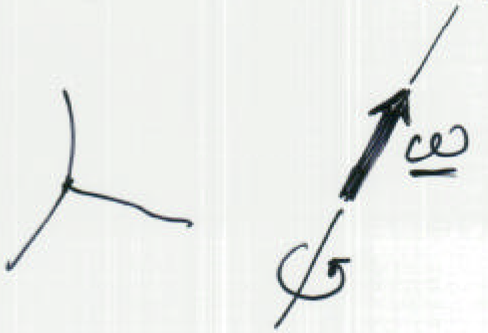
$$\underset{\dot{\underline{p}}}{\begin{bmatrix} \dot{x} \\ \hline \dot{\alpha} \end{bmatrix}} = \underset{6\times 6}{\begin{bmatrix} \boxed{J_L} \\ \hline \boxed{J_A} \end{bmatrix}} \underset{\dot{\underline{q}}}{\begin{bmatrix} 6\times 1 \end{bmatrix}}$$

$$J = \left[\frac{J_L}{J_A} \right]$$





VELOCITA' ANGOLARE



$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

VELOCITA' ANGOLARE & ROTAZIONI

$$R^T R = I \qquad R(t)R^T(t) = I$$

$$\underbrace{\frac{dR}{dt}R^T}_{S} + \underbrace{R\frac{dR^T}{dt}}_{S^T} = 0$$

$$S + S^T = 0$$

$$\boxed{S = -S^T}$$

ANTI SIMMETRICA

$$\boxed{\begin{matrix} M = M^T \\ \text{SIMMETRICA} \end{matrix}}$$

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$S = -S^T = -\begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$S(\underline{v})$ — data S posso estrarre v / dato v posso costruire S

$$S(\underline{v}) = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0 & 37 & 12 \\ \cdot & 0 & -6 \\ \cdot & \cdot & 0 \end{bmatrix} \quad \underline{v} \rightarrow = \begin{bmatrix} 6 \\ 12 \\ -37 \end{bmatrix}$$

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$$\frac{dR}{dt} R^T = S(\underline{v})$$

$$\boxed{\frac{dR}{dt} = S(v) R}$$

variazione istantanea dell'assetto

Cos'è $\underline{v}$?

dato:

$R(\underline{u}, \theta(t))$ cos'è

$\dot{R}$? $= S(\underline{\omega}(t))\, R(\underline{u}, \theta(t))$

$$\boxed{R(\underline{u}, \theta) = e^{S(\underline{u}, \theta)}}$$

$$\boxed{\underline{\omega} = \dot{\theta}(t)\, \underline{u}(t) + \sin\theta(t)\, \dot{\underline{u}}(t) + (1 - \cos\theta(t))\, S(\underline{u})\, \dot{\underline{u}}}$$

solo se $\dot{u} = 0$ cioè asse costante

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Riferimento - i - esimo

$\underline{\omega} \rightarrow \left[\omega_j^i(P)\right]_k$ Rappresentazione

Riferimento j - esimo

$\omega_i^i \equiv 0 \quad \omega_j^i + \omega_e^j$

$= \omega^i$