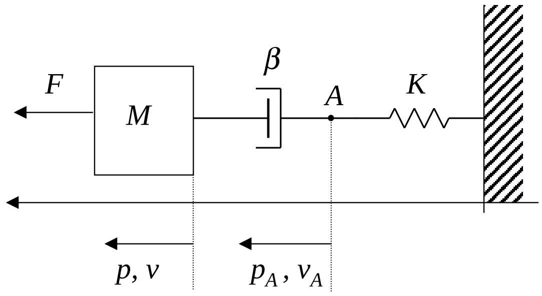
18AKSOA - CONTROLLI AUTOMATICI

I esercitazione presso il LAIB

Esercizio #1.a: simulazione della risposta di un sistema meccanico
Si consideri il seguente sistema dinamico meccanico SISO LTI a tempo continuo:



che, scegliendo come variabili di ingresso $u = [F]$, stato $x = [x_1,\ x_2]^T = [v,\ p_A]^T$ ed uscita $y = [\ v\ ]$, ha la seguente rappresentazione in variabili di stato:

$$\begin{cases} \dot{x}_1 = -\dfrac{K}{M}x_2 + \dfrac{1}{M}u \\ \dot{x}_2 = x_1 - \dfrac{K}{\beta}x_2 \end{cases}$$
$$y = x_1$$

dove M è la massa del corpo puntiforme, K è la costante di elasticità della molla, β è il coefficiente di attrito viscoso dello smorzatore, F è la forza applicata alla massa.

Assumendo $M = 0.2\,\text{kg}$ ed $F(t) = F_0 \cos(\omega_0 t)\,\text{N}$, si simuli il comportamento del sistema nei seguenti casi:

1) l'ingresso $F(t)$ è un gradino unitario ($F_0 = 1\,\text{N}$, $\omega_0 = 0\,\text{rad/s}$), mentre i valori numerici degli altri parametri e dello stato iniziale sono:

   1.a) $\beta = 0.1\,\text{Ns/m}$, $K = 2\,\text{N/m}$, $x(t=0) = x_0 = [0,\ 0]^T$;
   1.b) $\beta = 0.01\,\text{Ns/m}$, $K = 2\,\text{N/m}$, $x(t=0) = x_0 = [0,\ 0]^T$;
   1.c) $\beta = 10\,\text{Ns/m}$, $K = 20\,\text{N/m}$, $x(t=0) = x_0 = [0,\ 0]^T$;
   1.d) $\beta = 0.1\,\text{Ns/m}$, $K = 2\,\text{N/m}$, $x(t=0) = x_0 = [0,\ 0.2]^T$;

2) l'ingresso $F(t)$ è di tipo sinusoidale con $F_0 = 1$ N ed $\omega_0 = 4\,\text{rad/s}$, mentre i valori numerici degli altri parametri e dello stato iniziale sono:

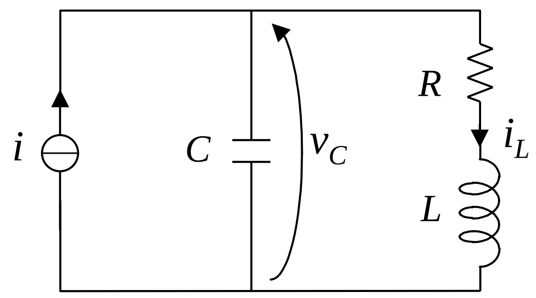
   2.a) $\beta = 0.1\,\text{Ns/m}$, $K = 2\,\text{N/m}$, $x(t=0) = x_0 = [0,\ 0]^T$;
   2.b) $\beta = 0.01\,\text{Ns/m}$, $K = 2\,\text{N/m}$, $x(t=0) = x_0 = [0,\ 0]^T$;
   2.c) $\beta = 10\,\text{Ns/m}$, $K = 20\,\text{N/m}$, $x(t=0) = x_0 = [0,\ 0]^T$;
   2.d) $\beta = 0.1\,\text{Ns/m}$, $K = 2\,\text{N/m}$, $x(t=0) = x_0 = [0,\ 0.2]^T$.

Tracciare i grafici degli andamenti dell'evoluzione temporale degli stati e dell'uscita.

Comandi MATLAB da prendere in considerazione: `ss`, `lsim`

Esercizio #1.b: simulazione della risposta di un sistema elettrico

Si consideri il seguente sistema dinamico elettrico SISO LTI a tempo continuo:



che, scegliendo come variabili di ingresso $u = [\, i \,]$, stato $x = [x_1,\ x_2]^T = [v_C,\ i_L]^T$ ed uscita $y = [v_C]$, ha la seguente rappresentazione in variabili di stato:

$$\begin{cases} \dot{x}_1 &= -\dfrac{1}{C}x_2 + \dfrac{1}{C}u \\ \dot{x}_2 &= \dfrac{1}{L}x_1 - \dfrac{R}{L}x_2 \\ y = x_1 \end{cases}$$

Assumendo $C = 0.2\,\mathrm{F}$ ed $i\,(t) = i_0 \cos(\omega_0 t)\,\mathrm{A}$, si simuli il comportamento del sistema nei seguenti casi:

1) l'ingresso $i\,(t)$ è un gradino unitario ($i_0 = 1\,\mathrm{A}$, $\omega_0 = 0\,\mathrm{rad/s}$), mentre i valori numerici degli altri parametri e dello stato iniziale sono:
   - 1.a) $R = 10\,\Omega$, $L = 0.5\,\mathrm{H}$, $x\,(t = 0) = x_0 = [0,\ 0]^T$;
   - 1.b) $R = 100\,\Omega$, $L = 0.5\,\mathrm{H}$, $x\,(t = 0) = x_0 = [0,\ 0]^T$;
   - 1.c) $R = 0.1\,\Omega$, $L = 0.05\,\mathrm{H}$, $x\,(t = 0) = x_0 = [0,\ 0]^T$;
   - 1.d) $R = 10\,\Omega$, $L = 0.5\,\mathrm{H}$, $x\,(t = 0) = x_0 = [0,\ 0.2]^T$;
2) l'ingresso $i\,(t)$ è di tipo sinusoidale con $i_0 = 1\,\mathrm{A}$ ed $\omega_0 = 4\,\mathrm{rad/s}$, mentre i valori numerici degli altri parametri e dello stato iniziale sono:
   - 2.a) $R = 10\,\Omega$, $L = 0.5\,\mathrm{H}$, $x\,(t = 0) = x_0 = [0,\ 0]^T$;
   - 2.b) $R = 100\,\Omega$, $L = 0.5\,\mathrm{H}$, $x\,(t = 0) = x_0 = [0,\ 0]^T$;
   - 2.c) $R = 0.1\,\Omega$, $L = 0.05\,\mathrm{H}$, $x\,(t = 0) = x_0 = [0,\ 0]^T$;
   - 2.d) $R = 10\,\Omega$, $L = 0.5\,\mathrm{H}$, $x\,(t = 0) = x_0 = [0,\ 0.2]^T$.

Tracciare i grafici degli andamenti dell'evoluzione temporale degli stati e dell'uscita.

Comandi MATLAB da prendere in considerazione: `ss`, `lsim`

- **SS** Create state-space models or convert LTI model to state space.

  You can create a state-space model by:

  `SYS = SS(A,B,C,D)` Continuous-time model
  `SYS = SS(A,B,C,D,T)` Discrete-time model with sampling time `T` (Set `T=-1` if undetermined)
  `SYS = SS` Default empty state-space model
  `SYS = SS(D)` Static gain matrix
  `SYS = SS(A,B,C,D,LTISYS)` State-space model with LTI properties inherited from the LTI model `LTISYS`.

  All the above syntaxes may be followed by Property/Value pairs. (Type `help ltiprops` for details on assignable properties). Setting `D=0` is interpreted as the zero matrix of adequate dimensions. The output `SYS` is an `SS` object.

  `SYS = SS(SYS)` converts an arbitrary LTI model `SYS` to state space, i.e., computes a state-space realization of `SYS`.

- **LSIM** Simulation of the time response of LTI systems to arbitrary inputs.

  `LSIM(SYS,U,T)` plots the time response of the LTI model `SYS` to the input signal described by `U` and `T`. The time vector `T` consists of regularly spaced time samples and `U` is a matrix with as many columns as inputs and whose `i`-th row specifies the input value at time `T(i)`. For instance,

  `t = 0:0.01:5; u = sin(t); lsim(sys,u,t)`

  simulates the response of `SYS` to u(t) = sin(t) during 5 seconds.
  In discrete time, `U` should be sampled at the same rate as the system (`T` is then redundant and can be omitted or set to the empty matrix).
  In continuous time, the sampling period `T(2)-T(1)` should be chosen small enough to capture the details of the input signal. The time vector `T` is resampled when intersample oscillations may occur.

  `LSIM(SYS,U,T,X0)` specifies an additional nonzero initial state `X0` (for state-space systems only).

  `LSIM(SYS1,SYS2,...,U,T,X0)` simulates the response of multiple LTI systems `SYS1`, `SYS2`,... on a single plot. The initial condition `X0` is optional. You can also specify a color, line style, and marker for each system, as in

  `lsim(sys1,'r',sys2,'y--',sys3,'gx',u,t)`.

  When invoked with left hand arguments,

  `[Y,T] = LSIM(SYS,U,...)`

  returns the output history `Y` and time vector `T` used for simulation. No plot is drawn on the screen. The matrix `Y` has `LENGTH(T)` rows and as many columns as outputs in `SYS`.

  For state-space systems,

  `[Y,T,X] = LSIM(SYS,U,...)`

  also returns the state trajectory `X`, a matrix with `LENGTH(T)` rows and as many columns as states.

Esercizio #2: calcolo di funzioni di trasferimento

Si calcolino le funzioni di trasferimento dei sistemi dinamici precedentemente considerati.

Comandi MATLAB da prendere in considerazione: `ss2tf`

- **SS2TF** State-space to transfer function conversion.

  `[NUM,DEN] = SS2TF(A,B,C,D,iu)` calculates the transfer function:

  $$\mathtt{H(s)} = \frac{\mathtt{NUM(s)}}{\mathtt{DEN(s)}} = \mathtt{C(sI-A)}^{-1}\mathtt{B+D}$$

  of the system: $\dot{x} = Ax + Bu$, $y = Cx + Du$ from the `iu`'th input. Vector `DEN` contains the coefficients of the denominator in descending powers of s. The numerator coefficients are returned in matrix `NUM` with as many rows as there are outputs y.

Esercizio #3: calcolo analitico di risposte nel tempo di sistemi dinamici mediante antitrasformata di Laplace

Si calcolino analiticamente, per condizioni iniziali nulle, le risposte dei sistemi dinamici precedentemente considerati ai seguenti ingressi:

- gradino di ampiezza $u_0 : u(t) = u_0 \cdot \varepsilon(t)$
- rampa unitaria $u(t) = t \cdot \varepsilon(t)$
- coseno di ampiezza u_0 e di pulsazione 4 rad/s: $u(t) = u_0 \cos(4t) \cdot \varepsilon(t)$

Comandi MATLAB da prendere in considerazione: `tf`, `tfdata`, `residue`

- **TF** Creation of transfer functions or conversion to transfer function.

  You can create SISO or MIMO transfer functions by

  `SYS = TF(NUM,DEN)` Continuous-time model `NUM(s)./DEN(s)`
  `SYS = TF(NUM,DEN,T)` Discrete-time model `NUM(z)./DEN(z)` with sampling time `T` (Set `T=-1` if undetermined)
  `SYS = TF` Default empty transfer function
  `SYS = TF(M)` Static gain matrix
  `SYS = TF(NUM,DEN,LTISYS)` Transfer function with LTI properties inherited from the LTI model `LTISYS`.

  All the above syntaxes may be followed by Property/Value pairs. (Type `help ltiprops` for details on assignable properties). The output `SYS` is a `TF` object.

  By default, transfer functions are displayed as functions of 's' or 'z'. Alternatively, you can set the variable name to 'p' (continuous time) and 'z^{-1}' or 'q' (discrete time) by modifying the 'Variable' property.

For SISO models, NUM and DEN are row vectors listing the numerator and denominator coefficients in

- descending powers of s or z by default
- ascending powers of q = z^{-1} if 'Variable' is set to 'z^{-1}' or 'q' (DSP convention).

For MIMO models with NU inputs and NY outputs, NUM and DEN are NY-by-NU cell arrays of row vectors where NUM{i,j} and DEN{i,j} specify the transfer function from input j to output i. For example,

```
tf( -5 ; [1 -5 6] , [1 -1] ; [1 1 0])
```

specifies the two-output/one-input system

```
[ -5 /(s-1) ]
[(s^2-5s+6)/(s^2+s)]
```

SYS = TF(SYS) converts an arbitrary LTI model SYS to transfer function format. The output SYS is a TF object.

SYS = TF(SYS,'inv') uses a fast algorithm for state-space SYS, but is typically less accurate for high-order systems.

- ## **TFDATA** Quick access to transfer function data.

[NUM,DEN] = TFDATA(SYS) returns the numerator(s) and denominator(s) of the transfer function SYS. NUM and DEN are cell arrays with as many rows as outputs and as many columns as inputs, and their (I,J) entries specify the transfer function from input J to output I. SYS is first converted to transfer function if necessary.

[NUM,DEN,TS,TD] = TFDATA(SYS) also returns the sample time TS and input delays TD. For continuous systems, TD is a vector with one entry per input channel. For discrete systems, TD is the empty matrix [].

For SISO systems, the convenience syntax [NUM,DEN] = TFDATA(SYS,'v') returns the numerator and denominator as row vectors rather than cell arrays.

- ## **RESIDUE** Partial-fraction expansion (residues).

[R,P,K] = RESIDUE(B,A) finds the residues, poles and direct term of a partial fraction expansion of the ratio of two polynomials B(s)/A(s). If there are no multiple roots,

$$\frac{B(s)}{A(s)} = \frac{R(1)}{s - P(1)} + \frac{R(2)}{s - P(2)} + \frac{R(n)}{s - P(n)} + K(s)$$

Vectors B and A specify the coefficients of the numerator and denominator polynomials in descending powers of s. The residues are returned in the column vector R, the pole locations in column vector P, and the direct terms in row vector K. The number of poles is n = length(A)-1 = length(R) = length(P). The direct term coefficient vector is empty if length(B) < length(A), otherwise length(K) = length(B)-length(A)+1.

If P(j) = ... = P(j+m-1) is a pole of multiplicity m, then the expansion includes terms of the form

$$\frac{R(j)}{s - P(j)} + \frac{R(j+1)}{(s - P(j))^2} + \frac{R(j+m-1)}{(s - P(j))^m}$$

[B,A] = RESIDUE(R,P,K), with 3 input arguments and 2 output arguments, converts the partial fraction expansion back to the polynomials with coefficients in B and A.

Warning: Numerically, the partial fraction expansion of a ratio of polynomials represents an ill-posed problem. If the denominator polynomial, A(s), is near a polynomial with multiple roots, then small changes in the data, including roundoff errors, can make arbitrarily large changes in the resulting poles and residues. Problem formulations making use of state-space or zero-pole representations are preferable.