System identification, Estimation and Filtering

Exercise 1

Hair dryer model identification (real data).

Consider an hair dryer whose input is the electric power and output is the air temperature.

Problem:
1) Identify from experimental data several ARX, ARMAX and OE models of different orders for the hair dryer.
2) Select the "best" model among the ones identified at step 1. Use the following criteria/methods for the selection:
- AIC
- MDL
- best FIT, where the FIT index is defined as

$$FIT = 1 - \frac{\sqrt{\sum_{t=1}^{N} \left(y(t) - \widehat{y}(t)\right)^2}}{\sqrt{\sum_{t=1}^{N} \left(y(t) - m_y\right)^2}}, \; m_y = \text{ sample mean of } y$$

- cross validation
- residual analysis.

Main steps:

1) Open the *Ident GUI* by means of the *Matlab* command *ident*.
2) Import the dryer data (*Import data/Example*).
3) Remove means.
4) Partition the whole data set in two subsets: estimation data set (ES) and validation data set (VS).
5) Insert ES as the working data and VS as the validation data in the *Ident GUI*.
6) Perform the order selection (*Linear Parametric Models*) using an ARX structure and considering the AIC, MDL and best FIT criteria.
7) Identify several models of different orders using the following structures:
- ARX(na,nb,nk)
- ARMAX(na,nb,nc,nk)
- OE(nb,nf,nk)

where na, nb, and nk have been selected at step 6.
7) Compare the identified models on the set VS considering the best FIT index and the residual analysis.
8) Select the "best" model.

Exercise 2

Parameter convergence in ARX model identification (simulated data)

Consider the following ARX(1,2,1) system:

$$y(t) = -0.93y(t-1) + 1.5u(t-1) - 3u(t-2) + e(t)$$

where $e(t) \sim WN(0,9)$.

Problem:
1) Supposing that the parameter vector $\theta_o = [0.93, 1.5, -3]^T$ is unknown, derive the least-squares estimate $\widehat{\theta}_N$ of θ_o using N data.
2) Considering increasing values of N, verify the asymptotic convergence of the least-squares estimate $\widehat{\theta}_N$ to the true parameter vector θ_o:

$$\widehat{\theta}_N \underset{N\to\infty}{\longrightarrow} \theta_o.$$

Main steps:

1) Create a Matlab script for the simulation of the ARX(1,2,1) system:
- Use a *for* loop with $N = 1 : T$.
- Both standard and recursive least-squares can be used for identification (the latter are lighter from a computational standpoint).
- Use the command *randn* to generate both the input u and the noise e.
2) At each step N of the *for* loop:
- Derive an estimate $\widehat{\theta}_N$ using the data $y(t)$ and $u(t)$ with $t \leq N$.
- Insert the estimate $\widehat{\theta}_N$ in the N-th column of a $3 \times T$ matrix P.
3) Plot P' in function of N.