System identification, Estimation and Filtering

Problem 1

Parametric estimation of a resistance from current-voltage data.

Consider a resistor described by the following current-voltage relation:

$$V = Ri + \eta$$

where R is the resistance, i is the current in the resistor, V is the voltage across the resistor terminals, and η is a white noise with zero mean value. Suppose that the following data are available:

- $[i_1\ i_2\ \ldots\ i_N]^T$: $N = 20$ current measurements equispaced in the interval $[1, 10]$ *Ampere*;
- $[V_1\ V_2\ \ldots\ V_N]^T$: $N = 20$ noise-corrupted voltage measurements corresponding to the above current values.

Problem: Compute the following estimates of the resistance R:

- Sample Mean estimate $\widehat{R}_N$.
- Least Squares estimate $\widehat{R}_{LS}$.
- Gauss-Markov estimate $\widehat{R}_{GM}$.
- Bayesian estimate $\widehat{R}_B$.

Evaluate the quality of the obtained estimates by comparing on a plot the measured and "predicted" voltage.

Main steps:

(1) Load the data from the file *resistor_ data_ 1.mat* (*load* Matlab command).

(2) Plot on a figure the measured data (*plot* Matlab command).

(3) Compute the Sample Mean estimate $\widehat{R}_N$. Plot in the figure generated at step (2) the estimated resistor characteristic $\widehat{V}_N = \widehat{R}_N i$.

(4) Compute the Least Squares estimate $\widehat{R}_{LS}$ (*mldivide* Matlab command or "\" Matlab command). Plot in the figure generated at step (2) the estimated resistor characteristic $\widehat{V}_{LS} = \widehat{R}_{LS} i$.

(5) Compute the Gauss-Markov estimate $\widehat{R}_{GM}$ considering the disturbance variance matrix $\Sigma_\eta = \sigma_\eta^2 I_N$, where $\sigma_\eta = 5$ *Volt* and I_N is the identity matrix. Plot in the figure generated at step (2) the estimated resistor characteristic $\widehat{V}_{GM} = \widehat{R}_{GM} i$.

(6) Compute the Bayesian estimate as

$$\widehat{R}_B = \overline{R} + \Sigma_{RV}\Sigma_{VV}^{-1}(V - \overline{V})$$

where:

- $\overline{R} = E[R]$ is the mean value of R. This mean value is assumed a priori (try with a value around 3).
- $\overline{V} = \overline{R}i$.
- $\Sigma_{RR} = E[(R-\overline{R})(R-\overline{R})]$ = variance of R. This variance is assumed a priori (try with the following values: $10, 1, 0.1, 0.01$).
- $\Sigma_{RV} = E[(R-\overline{R})(V-\overline{V})^T] = E[(R-\overline{R})(Ri+\eta-\overline{R}i)^T] = E[(R-\overline{R})(Ri-\overline{R}i)^T] = \ldots = \Sigma_{RR} i^T$.
- $\Sigma_{VV} = E[(V-\overline{V})(V-\overline{V})^T] = \ldots = \Sigma_{RR} ii^T + \Sigma_\eta$, where $\Sigma_\eta = \sigma_\eta^2 I_N$.

Plot in the figure generated at step (2) the estimated resistor characteristic $\widehat{V}_B = \widehat{R}_B i$.

(7) Repeat the steps (1)-(6) using the data stored in the file *resistor_ data_ 2.mat*. In steps (5) and (6), use a variance matrix Σ_η consistent with the data.

Problem 2

Estimation of the parameters of an auto-regressive system from output data.

Consider the following auto-regressive system:

$$y(t) = a_1 y(t-1) + a_2 y(t-2) + b + \eta(t)$$

where $\eta(t)$ is a white noise with zero mean value and variance $\sigma_\eta^2 = 40$. Suppose that a set of $L = 500$ data has been generated by this system.

Problem: Estimate the parameters a_1, a_2 and b by means of Least Squares. Compare on a plot the measured and predicted output.

Main steps:
(1) Load the data from the file *AR_data.mat*.
(2) Define the output vector y and the matrix Φ according to

$$y = \begin{bmatrix} y(3) \\ \vdots \\ y(L) \end{bmatrix} \quad \Phi = \begin{bmatrix} y(2) & y(1) & 1 \\ \vdots & \vdots & \vdots \\ y(L-1) & y(L-2) & 1 \end{bmatrix}.$$

The measurement equation is $y = \Phi\theta + \eta$ where $\theta = [a_1 \ a_2 \ b]^T$.
(3) Compute the Least Squares estimate $\widehat{\theta}_{LS}$ of θ.
(4) Compute the predicted output as $y_{pred} = \Phi\widehat{\theta}_{LS}$. Plot on a figure y and y_{pred}.