Problem #1: the input-output measurements of a SISO dynamic system $S_1$ to be modeled have been collected in the MATLAB data1.mat file.

1) Identify ARX, ARMAX and OE models of different orders and delays, using only a part of the experimental data and looking for models that guarantee satisfactory characteristics of whiteness of the residuals associated to this first dataset.

2) Compare the identified models on a different set of data not used for identification. To assess the model quality, minimize the Root Mean Square Error $RMSE = \sqrt{\frac{1}{N-N_0} \sum_{t=N_0+1}^{N} [y(t) - \hat{y}(t)]^2}$, where $y(t) =$ measured output, $\hat{y}(t) =$ simulated (or predicted) output and $N_0$ is a suitable time instant after which the transient is past. Choose the best trade-off between $RMSE$ and model complexity: $n = n_a + n_b$ (for ARX models), $n = n_a + n_b + n_c$ (for ARMAX models) and $n = n_f + n_b$ (for OE models), if the model is used for prediction purposes; $n = n_a$ (for ARX and ARMAX models) and $n = n_f$ (for OE models), if the model is used for simulation purposes.

3) Using all the experimental data, estimate the parameters of an ARX(3,3,1) model by means of the standard Least-Squares algorithm and compare:

- the Estimate Uncertainty Intervals $EUI^2$ and, if possible, the Parameter Uncertainty Intervals $PUI^2$, assuming that the output measurements are corrupted by an energy-bounded noise whose $2$-norm is less than $4$;
- the Estimate Uncertainty Intervals $EUI^\infty$, assuming that the output measurements are corrupted by an amplitude-bounded noise whose $\infty$-norm is less than $0.1$.

Problem #2: consider the following LTI dynamic system $S_2$:

$$x(t+1) = Ax(t) + Bu(t) + v_1(t)$$
$$y(t) = Cx(t) + v_2(t)$$

where

$$A = \begin{bmatrix} 2.7258 & -1.2424 & 0.7577 \\ 2 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.026 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -0.45 & -0.05 \end{bmatrix}$$

$v_1(t)$ is a white noise with zero mean value and variance $V_1 = B_{v_1}B_{v_1}^T$, $B_{v_1} = 0.05 \begin{bmatrix} 0.026 & 0 & 0 \end{bmatrix}^T$; $v_2(t)$ is a white noise with zero mean value and variance $V_2 = 0.0004$ and $u(t)$ is a suitable input signal whose values have been saved in the MATLAB data2.mat file. The noises $v_1(t)$ and $v_2(t)$ are uncorrelated. Assume that the initial state $x(1)$ is a random vector with zero mean value and variance $P_1 = E[x(1)x(1)^T] = I_3$.

1) Design the steady-state Kalman filter $F_{\infty}$ and the dynamic Kalman 1-step predictor in predictor-corrector form $K_{pc}$.

2) Compare the state estimates provided by $F_{\infty}$ and $K_{pc}$ by means of graphical representations and evaluate the Root Mean Square Errors:

$$RMSE_k = \sqrt{\frac{1}{N' - N_0'} \sum_{t=N_0'+1}^{N'} [x_k(t) - \hat{x}_k(t)]^2}, \quad k = 1, \ldots, 3$$

where $\hat{x}_k(t)$ is the estimate of the state $x_k(t)$ and $N_0'$ is a suitable time instant after which the filter or predictor transient is past.